Revista Mexicana de Ingeniería Química

Vol. 9, No. 2 (2010) 219-229

SATURATED CONTROL DESIGN FOR BOUNDED SUBSTRATE REGULATION IN A CONTINUOUS ANAEROBIC BIOREACTOR

DISEÑO DE UN CONTROL SATURADO PARA LA REGULACION DE SUSTRATO EN UN BIOREACTOR ANAEROBICO CONTINUO

A. Morales-Díaz* and S. Carlos-Hernández

Cinvestav Saltillo, Carretera Saltillo-Monterrey Km. 13.5 Ramos Arizpe Coahuila, 25900, México

Received 11 of January 2010; Acceped 30 of June 2010

Abstract

The aim of the study presented in this paper is the synthesis of a new saturated control, which forces the substrate concentration to reach a pre-specified bounded region without performance degradation in the bioreactor operation. The proposed controller is designed to achieve performance despite input disturbance and kinetic parameters uncertainties; it is based on an exponential bounded function that allows a smooth and fast convergence by setting a tuning parameter in the exponential function. A stability analysis based on the Lyapunov theorem and also some tuning guidelines are presented. Finally, the applicability of the proposed controller is illustrated via numerical simulations in a model representing a CSTR in continuous operation mode.

Keywords: anaerobic bioreactor, saturated control, substrate regulation.

Resumen

En este artículo se presenta el diseño de un nuevo controlador saturado que permite la operación de un reactor anaeróbico dentro de una región previamente especificad, con lo cual se garantiza la estabilidad del proceso y se permite el cumplimiento de los estándares de calidad del agua residual tratada impuesta por las regulaciones ambientales. Dicha región de operación implica límites en la concentración de los sustratos del sistema y así mismo impone límites en la tasa de dilución que corresponde a la entrada de control. La propuesta de control contempla una función exponencial que permite manipular la convergencia del sistema dentro del área de operación de una manera suave y rápida. También se desarrolla un análisis de estabilidad empleando el teorema de Lyapunov y se proporciona una guía para la sintonización de los parámetros del controlador. El desempeño del sistema se muestra mediante simulaciones numéricas, para lo que se utiliza un modelo matemático que representa un reactor continuamente agitado en modo continuo.

Palabras clave: reactor anaeróbico, control saturado, regulación de sustrato.

1 Introduction

1.1 Some insights

Wastewater treatment processes are an alternative to reduce troubles caused by water pollution. In addition, if the treated water reaches the official standards, it can be reused in other applications: irrigation in agricultural activities, water gardens and parks and even for rivers recovering. Therefore, it is important to guarantee a high efficiency for the wastewater treatment plants. For these reasons, wastewater treatment is an active research topic nowadays.

Anaerobic digestion has high capacity to degrade complex substrates (containing large concentration of organic components), such as aromatic wastes and organic matter from disposal water (Jeris 1983; Fiestas, 1984; Olthof *et al.*, 1982; Olson *et al.*, 2005; Steyer *et al.*, 2006). Moreover, it is possible to apply anaerobic processes in order to reduce and to transform organic wastes from industrial and municipal effluents into a biogas which is composed mainly

* *Corresponding author. E-mail*: america.morales@cinvestav.edu.mx

of methane and carbon dioxide; then, biogas has an important potential to be used as an alternative energy source (Henze *et al.*, 1997). However, anaerobic digestion implies several problems to solve. In example, the sensitivity to variations in operating conditions (pH, overloads, temperature, substrate composition, etc.) can inhibit and even stop the process; the non-linear dynamics and the nonstationary nature of anaerobic digestion can be difficult to analyze and to control; the hydrodynamic behaviour of the reactor can induce transport phenomena hard to manipulate, then, to guarantee operational stability (Hill *et al.*, 1987), and to avoid the eventual breakdown of the anaerobic processes, the organic matter in the phase liquid must remain inside a predetermined set of values depending the reactor configuration and the characteristics of the wastewater to be treated (Ahring *et al.*, 1997). This entire situation represents important challenges associated with the regulation of the output substrate which is directly related to environmental regulations (Méndez-Acosta *et al.*, 2008). An additional issue is the use of the energy potential of biogas produced in anaerobic processes.

1.2 Control of the anaerobic digestion process

Anaerobic digestion degrades, in oxygen absence, organic matter into a gaseous mixture mainly composed by methane and carbon dioxide. Among several reactions that can be taken into consideration there are two mainly identified: acidogenesis (that produces volatile fat acids, VFA) and methanogenesis (that mainly produces methane). It is well known that in normal operating conditions the pH range is from 6 to 8, otherwise the bacterial growth can be stopped or inhibited causing an inadequate functioning of the bioreactor. For that reason, this process suffers from a lack of stability due to several factors, such as changes in the VFA concentration and pH. Moreover, a sudden change in the dilution rate causes either feed overload or feed under load, washing the reactor or changing substrate concentration, respectively; in both cases, the system becomes unstable.

For this reason, over the past decade, the regulation of the output organic wastes in anaerobic processes has been addressed by proposing many control techniques, the main goals are to keep certain operating variables which are indirect available such as the chemical oxygen demand (COD) and the biogas production at a predetermined value (Ahring *et al.*, 1997; Schügerl, 2001; Olsson *et al.*, 2005; Steyer *et al.*, 2006; Lindberg, 1995; Perrier *et al.*, 1993).

Some other works are related with the application of specific control techniques, for example in Ko, *et al.* (1982) the authors developed an adaptive control and identification of the dissolved oxygen process. Whereas, Bastin and Dochain (1990) developed strategies for on-line estimation and adaptive control of bioreactors. And also the extension of MIMO control for the design of adaptive controllers for non-linear stirred tank bioreactors (Dochain, 1991). In Mailleret *et al.* (2004), the authors developed an adaptive feedback of the gaseous flow-rate measurements; meanwhile in Mailleret *et al.* (2004), and Muller *et al.* (1996) a fuzzy control to manipulate dilution rate as a function of detected disturbances was made. In Sanchez *et al.* (2001), Carlos-Hernandez *et al.* (2007) a fuzzy control was also used to regulate bicarbonate inside bioreactors by combining different control actions as a function of the operating conditions, enhancing the processes performance.

Moreover classical proportional integral/ proportional integral derivative control (PI/PID) has also been applied with good results for the control of anaerobic digestion processes when there is a lack of knowledge about the mathematical model or even of the plant behaviour (Olson *et al.*, 2005; Steyer *et al.*, 2006), and its performance is highly dependent on tuning parameters. However, these systems are of nonlinear nature due to the biochemical reactions and also possess the presence of input constraints, which degraded the closed loop performance of the systems with this PI/PID classical control.

Despite the mathematical modelling to represent accurately the anaerobic process that helps in the control strategies development, its complex nonlinear and nonstationary nature impose some difficulties to regulate in a precise way the substrate concentration or even the biogas flow rate. Therefore, to preserve a stable operation and compliance of the stringent environmental polices, instead of treat the substrate regulation problem, a bounded control can be considered to keep inside of a pre-specified region the substrate concentration that can be inferred from an experimental steady state analysis. Moreover, due to physical and operational restrictions, the control input, commonly, the dilution rate must remain positive and cannot be arbitrarily high. In fact, the process capacity allows a maximum rate associated to the pumping mechanism and a minimum nonzero flow to maintain the bioreactor operation continuously. Related to this topic, a few authors have developed works for saturated control in bioreactors. For example, a robust regulation of a class of continuous bioreactors is proposed in Rapaport and Harmand (2002); the feedback output is constructed considering only partial measures of the states and uncertainties in the kinetics and exogenous disturbances. The main idea is to bound also the inputs and the uncertainties; therefore, the control input is saturated accordingly to the bounds in the uncertain kinetics. The proposed controller provides exponential stabilization in the worst case, however, a drawback of this work is that the stabilization of the nominal controller is discontinuous and jumps between the extreme bound values in the transient stage. Meanwhile, other authors designed a bounded output feedback controller law to make set point regulation of the methane gas flow rate (Antonelli *et al.*, 2003). The resulting controller is a nonlinear PI that yields set point regulation, rejects disturbances and preserves stability despite uncertainty on the kinetic. In other paper, an adaptive controller under uncertain parameters was developed to seek the maximum rate of gas production reachable (Marcos *et al.*, 2004). A reduced observer to estimate the substrate concentration in the bioreactor is designed. Moreover, a parameter adaptation is developed due to uncertainties in kinetics. To avoid the reactor washout, the magnitude of the controller and the parameters estimated are saturated outside the domain of interests. This work is supported by assuming knowledge of the kinetic coefficients, which is a mild assumption. Finally, other researches consider the regulation of a substrate combination by developing a saturated control Grognard and Bernard (2006), the main idea is to keep this combination inside a bounded region to avoid operational problems and maintain substrate degradation. The proposed controller allows the process to lead the substrate inside the bounded region; however, the control action presents oscillations which can induce performance degradation in the bioreactor operation, an undesirable situation.

1.3 This work

The first contribution of this paper is the synthesis of a new saturated control, which allows the process to enhance the influent treatment and to improve the disturbances rejection, since the objective is to keep the substrate concentrations in a bounded region. Moreover, the proposed controller is designed to achieve good performances despite input overloads and kinetic parameters uncertainties, and is based on exponential bounded function that allows a smooth and fast convergence by setting a tuning parameter in the exponential function.

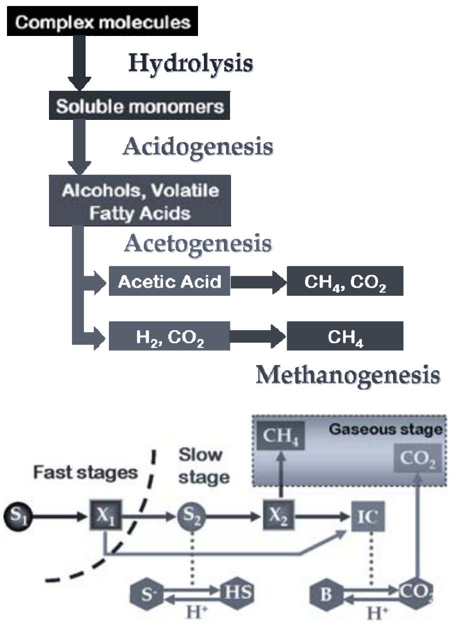
The second contribution of the paper concerns the stability properties of the anaerobic process with the proposed controller. The Lyapunov theorem is considered in order to find the operating conditions which guarantee the closed loop stability of the system. The applicability of the proposed controller is illustrated via numerical simulations in a model representing a CSTR in continuous operation mode.

The organization of this paper is as follows. In the next section, some preliminaries concerning the process model and assumptions on the bioreactor operation are presented in order to determine a working region that yields a stable performance in the bioreactor. Section 3 describes the proposed controller and a stability analysis; also some tuning guidelines are provided. Section 4 presents numerical simulations to test the behavior of the closed-loop system in face of different operating conditions. A comparison with another work (Grognard and Bernard, 2006) is included. After that, obtained results are discussed. Finally, in the last section 5 some relevant conclusions are stated.

2 Theoretical preliminaries

2.1 Process description

Anaerobic digestion is developed in four successive stages: hydrolysis, acidogenesis, acetogenesis, methanogenesis (Fig. 1a). The organic molecules are progressively degraded and the final product is a gas mixture which is mainly composed of methane and carbon dioxide. The stages can be classified with respect to the dynamics in fast and



a)

b)

Fig. 1: Schematic representation of anaerobic digestion. a) Successive stages, b) Functional diagram.

slow stages. The last one is the slowest and the most sensitive to variations on the operating conditions; then, it is considered as the limiting stage. Even if the dynamics of the other three stages are different, the global dynamic is faster than the methanogenesis; then, hydrolysis, acidogenesis and acetogenesis are grouped and considered as a fast stage. A functional scheme of anaerobic digestion is presented in Fig. 1b in order to illustrate this classification and the physic chemical equilibria, the materials conservation and the influence of pH.

On the other side, the hydrodynamic phenomena depend directly on the bioreactor. In this case, a completely stirred tank reactor in continuous mode is considered (Fig. 2). The input flow rate is equal to the output flow rate in order to keep the volume constant inside the reactor. The transport phenomena are then governed by the dilution rate. The advantage of this operation mode is that the wastewater can be fed continuously to the reactor until $t \to \infty$; in addition, the hydrodynamic behavior (dilution rate) is relatively easy to manipulate and to control. The main inconvenient of the continuous mode is that the modifications on the input flow rate have an influence on the residence time

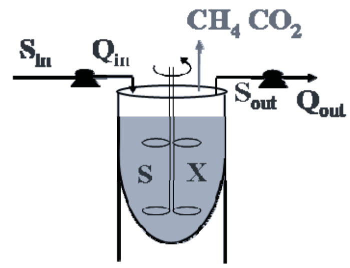


Fig. 2. Basic representation of a completely stirred tank reactor.

which can affect the wastewater treatment quality. For that reason, control strategies are required and in this paper a saturated controller is proposed to overcome this situation.

The usual measures online in this kind of process are: pH, production of methane and measures of carbon dioxide. On the other side substrate degradation which is related to the chemical oxygen demand (COD) is done off line by chemical analysis.

2.2 Mathematical model

The mathematical representation of the process described previously is composed of five algebraic equations (to model the physic chemical equilibria and the electroneutrality) and six differential equations (to model the process dynamics: bacteria growth, substrates degradation, inorganic carbon production and cations evolution). The complete model is shown in the set of equations (1) and (2).

$$
\begin{aligned}
HS + S^- - S_2 &= 0 \\
H^+S^- - K_a HS &= 0 \\
H^+B - K_b CO_{2d} &= 0 \\
B + CO_{2d} - C &= 0 \\
B + S^- - Z &= 0
\end{aligned}
\tag{1}
$$

where HS is non ionized acetic acid (mol/L), S^- ionized acetic acid (mol/L), H^+ ionized hydrogen (mol/L), B measured bicarbonate (mol/L), CO_{2d} dissolved carbon dioxide (mol/L), C inorganic carbon (mol/L), Z the total of cations (mol/L), K_a is an acid-base equilibrium constant, K_b is an equilibrium constant between B and CO_{2d}.

The differential equations define the state vector, $\xi = [X_1, X_2, S_1, S_2, C, Z]^T$

$$\begin{aligned}
\dot{X}_1 &= (\mu_1(S_1) - \alpha D)X_1 \\
\dot{X}_2 &= (\mu_2(S_2) - \alpha D)X_2 \\
\dot{S}_1 &= D(S_{1in} - S_1) - \kappa_1\mu_1(S_1)X_1 \\
\dot{S}_2 &= D(S_{2in} - S_2) + \kappa_2\mu_1(S_1)X_1 \\
&\quad -\kappa_3\mu_2(S_2)X_2 \\
\dot{C} &= D(C_{in} - C) - q_C(\xi) + \kappa_4\mu_1(S_1)X_1 \\
&\quad -\kappa_5\mu_2(S_2)X_2 \\
\dot{Z} &= D(Z_{in} - Z)
\end{aligned} \tag{2}$$

with

$$q_C(\xi) = \kappa_L a[C + S_2 - Z - K_H P_C(\xi)] \tag{3}$$

μ_1 is the growth rate (Haldane type) of X_1 (h^{-1}), μ_2 is the growth rate (Haldane type) of X_2 (h^{-1}), k_{d1} is the death rate of X_1 (mol/L), k_{d2} is the death rate of X_2 (mol/L), D_{in} is the dilution rate (h^{-1}), S_{in} the fast degradable substrate input (mol/L), S_{2in} the slow degradable substrate input (mol/L), C_{in} the inorganic carbon input (mol/L) and $R_1, ..., R_6$ are the yield coefficients.

2.3 Assumptions

To understand the interaction between the variables, the steady state is analyzed as follows. Let us to introduce three assumptions regarding the bioreactor (1), this will be used throughout the following sections.

(A.1) Considering the steady state in the bacterial growth rates we have that

$$\mu_1(S_1^*) = \mu_2(S_2^*) = \alpha D \geqslant 0 \tag{4}$$

the superscript (*) represents steady values.

(A.2) The functions $\mu_i(S_i)$ for $i = 1, 2$ are non-negative, bounded such that $\mu_i(0) = 0$, and $\mu_i(S_i) \leqslant \mu_{i\,\max}$.

(A.3) The input constraints D_{min} and D_{max} and the steady state in the substrates S_i^* are such that the following inequalities hold:

$$\begin{aligned}
&\forall S_i \in [0, S_{i,in}] \\
&0 < D_{\min} \leqslant \frac{\kappa_1\mu_1(S_1^*)X_1^*}{S_{1in} - S_1^*} < D_{\max} \\
&0 < D_{\min} \leqslant \frac{-\kappa_2\mu_1(S_1^*)X_1^* + \kappa_3\mu_2(S_2^*)X_2^*}{S_{2in} - S_2^*} < D_{\max}
\end{aligned} \tag{5}$$

Assumption (A.1) is standard and indicates that both kinetics and the input are bounded and smooth functions. Meanwhile (A.2) means that when substrate concentration is zero no bacterial activity is developed and there is a maximum production rate allowed. By assumption (A.3), there exists a substrate interval $[S_{i,I}\ , S_{i,II}\ ]$, such that $S_{i,I} \leq S_i \leq S_{i,II}$ where the inequalities (4) are satisfied for all $S_i \in [S_{i,I}, S_{i,II}] \times [0, S_{i,in}]$, and comprises the equilibrium point ξ^*. This assumption can be regarded as a kind of feasibility condition in the open loop system. Indeed, it implies that the static input D^* corresponding to ξ^* belongs to the interval of the input constraints and it might hold only for a certain range of the manipulated variable where the bioreactor stays stable.

2.4 Relation between pH and D

As said before, pH is an important parameter for anaerobic processes and it must be fixed in a desired level. A specific value of pH, named pH_{in}, implies a specific value of the process inputs. In the case of the saturated control, it is important to deduce relations between the desired pH and the input control in order to determine the minimal and maximal values of the dilution rate. To obtain such relations, the next procedure is proposed.

1. The equilibrium equation of X_2^* (second differential equation) is considered because the biomass growth rate μ_2 depends on S_2 which is affected by pH.

$$\mu_2(S_2^*) = \alpha D = \frac{\mu_{2\,\max}S_2^*}{k_{s2} + S_2^* + \frac{(S_2^*)^2}{k_{i2}}} \tag{6}$$

2. From the equilibrium equations for S_2^* (first and second algebraic equations), the next expression is deduced.

$$S_2^* = SH^*\left(1 + \frac{k_a}{H^+}\right) \tag{7}$$

Replacing Eq. (7) in (6) and considering $\mathrm{pH} = -\log_{10}(H^+)$ an equation for D in function of pH is obtained.

$$\alpha D = \frac{\mu_{2\,\max}SH^*\left(1 + \frac{k_a}{10^{-pH}}\right)}{k_{s2} + SH^*\left(1 + \frac{k_a}{10^{-pH}}\right) + \frac{SH^{*2}\left(1+\frac{k_a}{10^{-pH}}\right)^2}{k_{i2}}} \tag{8}$$

With this expression it is possible to determine the lower and upper bounds of the control action considering the desired interval of pH. In this paper, the interval pH=[6, 8] is considered since it is an optimal interval for process operation.

3 Saturated control design

In the bioreactor inlet the concentration of contaminants is high; a desirable operation is to diminish this concentration in the outlet flow. From a control point of view, this means that the output substrate concentration must be regulated. It is known that the dilution rate is the control input capable to modify the outlet substrates. Then, the foremost contribution of this study deals with the design of a feedback control strategy that forces the substrate concentrations to reach a bounded region defined through the input constraints and the inequalities (4). The control structure is based on an exponential relation between the bounded substrate and the dilution rate as explained in next lines.

The rate change of substrate S_1 depends on the input substrate, S_{1in}, and on the degradation of this one by the biomass action. Also the substrate S_2 depends on S_{2in} and on the transformation of S_1 into S_2. The rate of this transformation is given by the parameter λ which is calculated as $\lambda \leq \kappa_2/\kappa_1$ and corresponds to the production ratio in the acidogenesis pathway (Carlos-Hernandez *et al.* (2007)).

In this paper, as in Grognard and Bernard (2006), a linear combination of the substrates accordingly to the input constraints is considered as the regulated variable and is described by the following equation:

$$S_\lambda = S_1 + \lambda S_2 \tag{9}$$

This definition of S_λ is related to the total COD which is associated with the pollution in the substrate.

The main idea of the control consists in reaching two goals: (i) to maintain substrates in a safe region that considers water treatment and (ii) to keep a stable operation in the bioreactor.

By considering the realistic assumption that not all S_1 is transformed to produce S_2, therefore $\lambda < \kappa_2/\kappa_1$ is taken. Now, in order to propose a control law, the dynamics of the regulation variable (8) is analyzed,

$$\dot{S}_\lambda = D(S_{\lambda in} - S_\lambda) + \lambda(\kappa_2 - \kappa_1)\mu_1(S_1)X_1 - \kappa_3\lambda\mu_2(S_2)X_2 \tag{10}$$

where $S_{\lambda in} = S_{1in} + \lambda S_{2in}$.

From the steady state analysis in (9), the next equation is obtained:

$$S_\lambda^* = S_{\lambda in} + \frac{\lambda(\kappa_2 - \kappa_1)\mu_1(S_1^*)X_1^* - \kappa_3\lambda\mu_2(S_2^*)X_2^*}{D^*} \tag{11}$$

From this, it can be concluded that in the steady state a unique value in the dilution rate D^* corresponds to S_λ^*. In addition, from assumption (A.3) the pair (S_λ^*, D^*) defines entirely the steady state set $\xi^* = [X_1^*, X_2^*, S_1^*, S_2^*, C^*, Z^*]^T$ in the bioreactor (1); that means, a unique value of each state variable is obtained for a specific (defined) value of (S_λ^*, D^*).

Accordingly to (4) the regulation variable S_λ is bounded as,

$$S_{\lambda\min} \leqslant S_\lambda \leqslant S_{\lambda\max} \tag{12}$$

or equivalently to (by using 10 and 3):

$$D_{\min} < \frac{\mu_1(S_1^*)}{|(S_\lambda^* - S_{\lambda in})|}\left[(\lambda\kappa_2 - \kappa_1)X_1^* - \kappa_3\lambda X_2^*\right] < D_{\max} \tag{13}$$

Moreover, to ensure a diminish in substrate concentration and considering (10) the upper bound in (11) is

$$S_{\lambda\max} < S_{\lambda in} \tag{14}$$

Then, the input constraints [D_{min} , D_{max}] imply a bounded interval in S_λ. To satisfy this condition, a feedback controller is proposed as follows,

$$D = D_{avg}\left[2 - \frac{2}{1 + e^{-\beta|S_\lambda|}}\right] + D_{\min} \tag{15}$$

where $D_{avg} = (D_{\max} - D_{\min})/\gamma$; and $\beta > 0$, $\gamma > 0$ are the adjusting parameters. The parameter β is responsible of the control rate convergence. Meanwhile, γ must be chosen as $0 \leq \gamma \leq 2$, to ensure that the limits in D are fulfilled.

The control proposed in this work, Eq. (15), is chosen in order to provide an exponential convergence with a free parameter (β); and to introduce the average value for the dilution rate this allows the user to resize the desired bounded region. The main difference with the work presented in Grognard and Bernard (2006) is the rate convergence which is imposed by the controller saturated function. Besides, only a bounded of the arithmetic average is considered.

Finally these authors used a particular version of the saturated function that seems to induce an oscillatory behavior.

By looking at (14) it can be inferred that for $0 \leq S_\lambda^* < S_{\lambda in}$, D^* is bounded in agreement with:

$$D_{\min} \leqslant D^* \leqslant \frac{1}{\gamma}\left[D_{\max} - D_{\min}(\gamma - 1)\right] \tag{16}$$

3.1 Stability analysis

The following candidate Lyapunov function is considerered for the control closed loop system (9), (14)

$$V = \frac{D^2}{2} \tag{17}$$

Taking the derivative of V, the next expression is deduced:

$$\begin{aligned} \dot{V} &= D\dot{D} \\ \dot{V} &= \left(D_{avg}\left[2 - \frac{2}{1+e^{-\beta|S_\lambda|}}\right] + D_{\min}\right) \\ &\quad \frac{-2\beta e^{-\beta|S_\lambda|}}{\left(1+e^{-\beta|S_\lambda|}\right)^2}\dot{S}_\lambda \end{aligned} \tag{18}$$

The first term in (17) corresponds to D, which for construction is positive and bounded (see 15), meanwhile because of $\beta > 0$, $\gamma > 0$ the time derivative of D remains negative, therefore to demonstrate convergence of the Lyapunov function proposed it is sufficient to prove that is positive. The dynamics (9) is provided by the mass balance equations that considers the substrate conversion S_λ to VFA and methane. Due $S_\lambda < S_{\lambda in}$ the first term in (9) stays positive. Meanwhile, the remaining terms for the conversion in substrates S_1 and S_2 to produce VFA and methane, respectively, also are positive. This is summarized as follows:

$$\begin{aligned} &S_{\lambda in} - S_\lambda > 0 \\ &(\lambda\kappa_2 - \kappa_1)\mu_1(S_1)X_1 - \lambda\kappa_3\mu_2(S_2)X_2 \geqslant 0 \end{aligned} \tag{19}$$

Therefore,

$$\dot{S}_\lambda \geqslant K_0 \tag{20}$$

where $K_0 > 0$, and (18) is

$$\dot{V} \leqslant -K_0\left(e^{-\beta|S_\lambda|}\right)D_{\min} \tag{21}$$

From (20) we can conclude the stability of the closed loop system with a convergence ratio of $K_0\beta$. This is stated in theorem 1.

Theorem 1. *Let (10) to be a non-linear system and (15) a control input; then, the closed loop system composed of (10) and (15) remains inside a pre-specified region given by the pair (S_λ, D) and the controller stabilizes the state vector ξ.*

In the case of regulation for substrate S_1 the following remark is considered.

Remark 1. *Regulation of S_1 can be considered in a similar way as in S_λ. Therefore, applying the same control (15) and by substituting S_1 instead of S_λ a similar result as in Theorem 1 can be provided.*

3.2 Tuning parameters

The tuning parameters β and γ in control (15) can be chosen as follows.

- For $\gamma = 1$, $D^* = D_{max}$ and for $\gamma = 2$, $D^* = 0.5[D_{max} - D_{min}]$, which is the arithmetic average. Therefore, γ must be chosen as $0 \leq \gamma \leq 2$, to ensure that the limits in D are fulfilled.
- To adjust β, the control dynamics stated in (18) is considered. Equation (21) shows that control convergence is achieved with rate $K_0\beta > 0$. Selection of large values in β implies a fast control convergence rate, which results in a poor performance and even causes instabilities. Instead, it is recommended to choose β near the natural time responses (dilution rate), i.e., $0 < \beta \leq D^*$.

4 Numerical results

Some numerical results are presented using the control (15) in the anaerobic bioreactor (1)-(2). The bioreactor parameters are obtained from Grognard *et al.* (2006), Bernard *et al.* (2001). The bounds in the dilution rate are calculated through (8).

For the initial conditions, model bounds and control parameters see Table 1. In Fig. 3 the designed control performance is shown. The proposed control fulfills the boundary requirements in a fast and smooth way. In fact, the dilution rate remains inside the $[D_{min}, D_{max}]$ interval. Besides, S_λ reaches its steady state with fast convergence inside the bounded region $S_{min} \leq S_\lambda \leq S_{max}$. Also, it can be noticed that

Table 1. Initial conditions, model bounds and control parameters.

$S_1(0)$=15	$S_2(0)$=15
$\chi_1(0) = 0.1$	$\chi_2(0) = 0.1$
$S_{max} = 4.6$	$S_{min} = 0.01$
$S_{1max} = 1.42$	$S_{1min} = 0.029$
$S_{2max} = 3.45$	$S_{2min} = 0$
$S_{1in} = 15$	$S_{2in} = 15$
$D_{min} = 0.01$	$D_{max} = 0.4$
$\lambda = 2.23$	$\alpha = 0.5$
$\beta = 0.01$	$\gamma = 1.5$

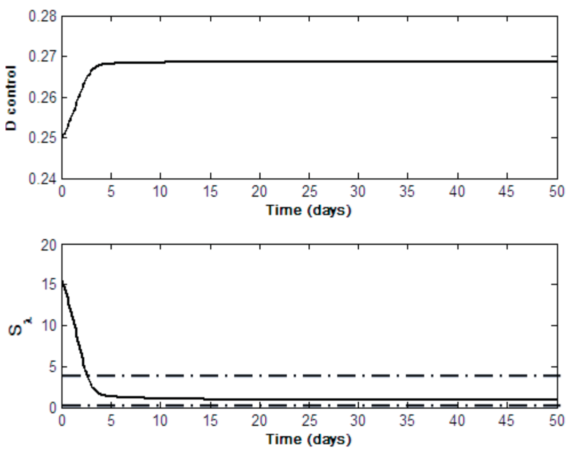


Fig. 3: Control action and regulated variable using the designed bounded control.

the substrate concentration is high in the reactor start up, which corresponds to a low bacterial growth rate. After some time, the bacterial concentration is increased and the pollution concentration is decreased to fulfill the boundaries.

For comparison purposes, Fig. 4 shows the control signal obtained using the proposed controller and that one with the controller developed on Grognard and Bernard (2006). The bounds considered are those reported in Grognard and Bernard (2006) and the values of γ and β remain constant. Both controllers meet the boundary requirements; however the second one has oscillations. This could cause instability problems in the bacterial growth. The controller designed in this paper shows a faster convergence than that of the controller of comparison.

To illustrate the performance of the controller (15), two step disturbances are induced in S_1 and S_2, one of +10 and other of +25 percent, respectively, at t = 50 days. This is shown in Fig. 5 for the dilution rate and for the S_λ. In face of this disturbance, the control is capable to remain the S_λ on the pre-specified bound region. Also, the steady state is reached in a fast way (20 days).

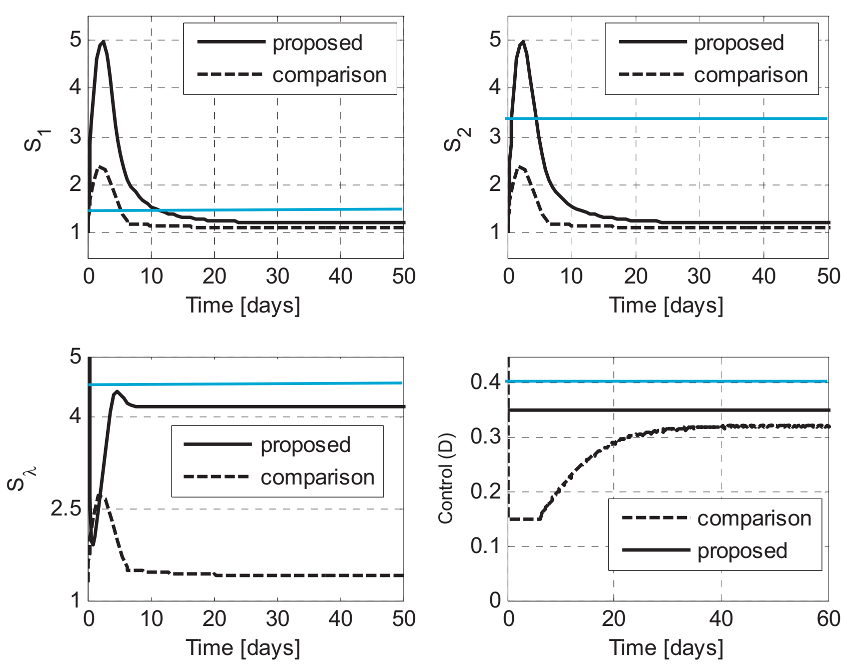


Fig. 4: Saturated control design for bounded substrate regulation in a continuous anaerobic bioreactor.

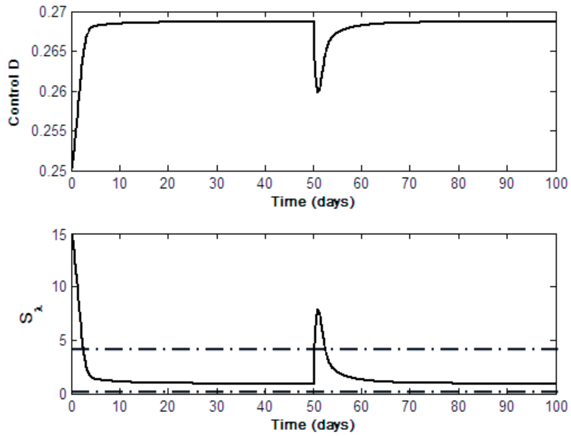


Fig. 5: Performance for disturbances in the inlet (at $t = 50$ day with amplitude of: $+5$ in both S_{1in} and S_{2in}).

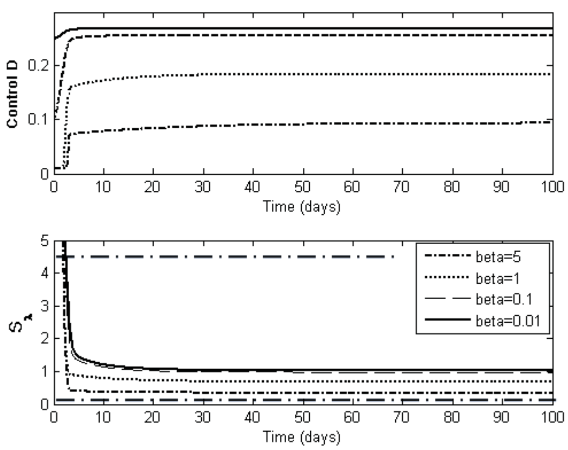


Fig. 6: Dilution rate and S_λ for several values of β.

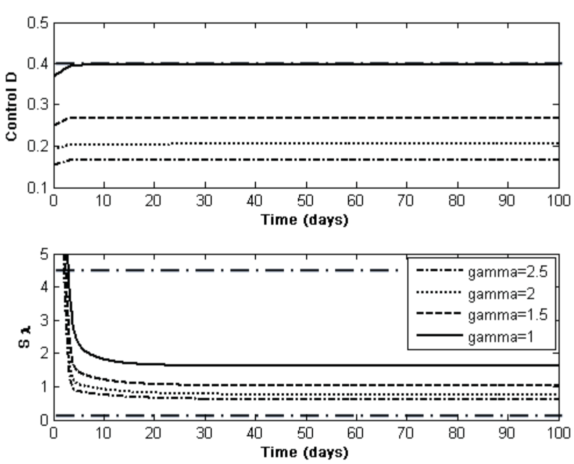


Fig. 7: Dilution rate and S_λ for several values of γ.

The controller performance for several tuning parameters β and γ is tested, see figs. 6 and 7. As it was expected, the simulations show that an increase of β maintains the bioreactor near its saturation limits, suitable values are those of the order of dilution time. A decrease in the parameter γ enlarges the bounded region in the controller; otherwise the controller action is shorten and increases vulnerability against disturbances.

Finally, using the best parameters tuned in the last simulations, uncertainties modeled on the substrates and on the biomass growth rates as white noise were considered. The same operating conditions considered previously are employed using the proposed control action, figs. 8 show the obtained results when noise (± 10 percent of μ_i) is included on the biomass growth rates.

The control action acts as a function of these noised operating conditions without reaching the saturation bounds. Furthermore, even if S_λ presents a noised behavior, the pres-specified bounded region is kept. There are only some impulses corresponding to the maximal effect of noise, where S_λ crosses the upper bound. Otherwise, the noise causes that S_λ remains near the boundaries inside the respective region; the parameter γ could be modified in order to enlarge the valid region of S_λ. Fig. 9 presents graphics for the dilution rate and S_λ when noise is included on the substrates (± 10 percent of the equilibrium values).

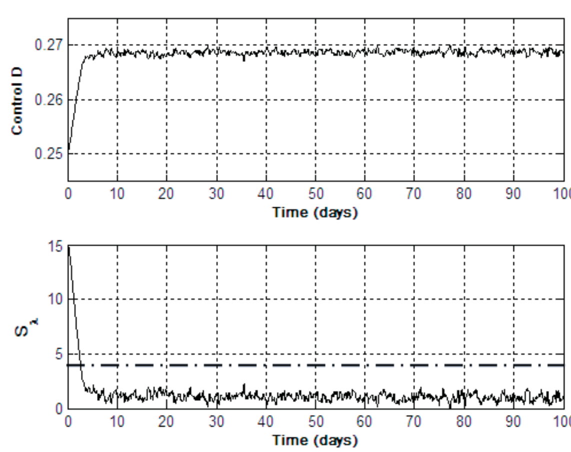


Fig. 8: Performance considering noise on growth rates (10% of μ_{imax}).

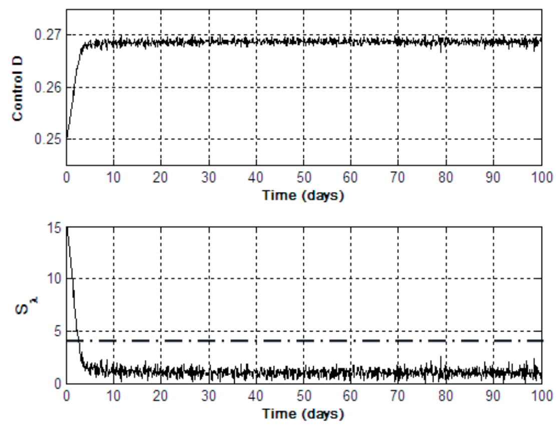


Fig. 9: Performance considering noise on substrates (10% of S_i^*).

The dilution rate remains between the bounded region without reaching saturation; furthermore, S_λ is always inside the valid region. For the maximal amplitude of the noise, S_λ is near to the upper bound. γ can be modified in order to enlarge or to move the bounded interval, as for previous simulations.

Conclusions

To keep the system in a stable region, bounds for the dilution rate were calculated by using the chemical equilibrium of hydrogen ions and the production of VFA. After that, a bounded control for the dilution rate in an anaerobic bioreactor was proposed. The total amount of COD was chosen as regulation variable and was driven inside the bounded region previously specified. A method to tune the parameters was also described and used to deduce the best values for tuning the controller. Several numerical simulations were performed in order to validate the proposed methodology controller.

The control shows a good performance. The regulated variable remains in the specified region even in face of disturbances on the input substrates and also considering noise on the biomass growth rates and the substrates. The control action fulfills the boundary conditions and the saturation values are not reached.

The capabilities of the controller were proven by changing the tuning parameters. As a future work the controller will be implemented in a real process.

References

Antonelli, R., J. Harmand, J.P. Steyer and A. Astolfi (2003). Set point regulation of an anaerobic digestion process with bounded output feedback. *IEEE Transactions on control systems technology 11*, 495-504.

Bastin, G. and D. Dochain (1990). *On-line estimation and adaptive control of bioreactors.* Elsevier, Amsterdam p. 379.

Bernard, O., Z. Hadj-Sadok, D. Dochain, A. Genovesi and J.P. Steyer (2001). Dynamic model development and parameter identification for an anaerobic wastewater treatment process. *Biotechnology and Bioengineering 75*, 424-438.

Carlos-Hernandez, S., E. N. Sanchez and J.-F. Beteau (2007). Intelligent control strategy for an anaerobic fluidized bed reactor. *10^{th} IFAC International Symposium on Computer Applications on Biotechnology 1*, 69-74.

Dochain, D. (1991). Design of adaptive controllers for non-linear stirred tank bioreactors: extension to the mimo situation. *Journal of Process Control 1*, 41-48.

Grognard, F. and O. Bernard (2006). Stability analysis of a wastewater treatment plant with saturated control. *Water Science & Technology 53*, 149-157.

Ko, K.Y-J, B.C. McInnis and G.C. Goodwin (1982). Adaptive control and identification of the dissolved oxygen process. *Automatica 18*, 727-730.

Lindberg, C.-F. (1995). *Control of wastewater treatment plants.* Licentiat Thesis, Institute of Technology Uppsala University, Sweden.

Mailleret, L., O. Bernard and J. Steyer (2004). Nonlinear adaptive control for bioreactors with unkown kinetics. *Automatica 40*, 1379-1385.

Marcos, N.I., M. Guay and D. Dochain (2004). Output feedback adaptive extremum seeking control of a continuous stirred tank bioreactor with Monod's kinetics. *Journal of Process Control 14*, 807-818.

Muller, A., S. Marsili-Libelli, A. Aivasidis, T. Lloyd, S. Kroner and C. Wandrey (1996). Fuzzy control of disturbances in a wastewater treatment process. *Water Research 31*, 3157-3167.

Perrier, M. and D. Dochain (1993). Evaluation of control strategies for anaerobic digestion process. *International Journal on Adaptive Control and Signal Processing 7*, 309-321.

Punal, A., J. Rodriguez, A. Franco, E. Carrasco, E. Roca and J. Lema (2001). Advanced monitoring and control of anaerobic wastewater treatment plants: Diagnosis and supervision by a fuzzy-based expert systems. *Water Science & Technology 43*, 191- 198.

Rapaport, A. and J. Harmand (2002). Robust regulation of a class of partially observed nonlinear continuous bioreactors. *Journal of Process Control 12*, 291-302.

Sanchez, E. N., J.-F. Beteau and S. Carlos-Hernandez (2001). Hierarchical fuzzy control for a wastewater anaerobic treatment plant. *IEEE International Conference on Systems Man and Cybernetics 5*, 3285-3290.

Vanrrolleghem, P.A. (1994). *On-line modelling and control of activated sludge process: development of an adaptive sensor*. PhD. Thesis Faculty of Agricultural and Applied Biological Sciences, University Gent, Belgium.